MATH 2230 HW7

Page 177

① If $f = u + iv$ and $g = e^f$, then

$$|g| = |e^f| = e^u \le e^{u_0} \quad \forall z \in \mathbb{C}.$$

Thus by Liouville thm, $g = e^f$ is a constant.

We see that $e^u \cos v$ and $e^u \sin v$ are constant.

By Cauchy Riemann equation, we see that u must be constant.

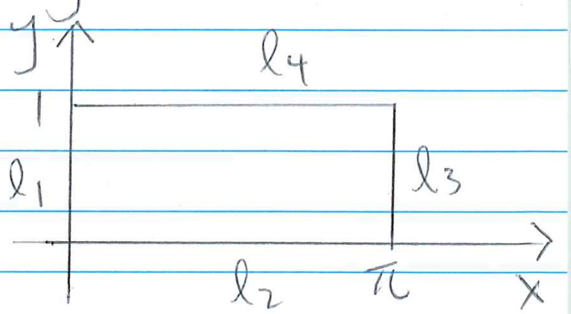
② Since $f \neq 0$ in R and f is analytic in the interior of R, $1/f$ is analytic in the interior of R. By maximum modulus principle and the continuity of f in R, $1/|f|$ attains its maximum value on the boundary of R. Therefore, $|f|$ attains a minimum value m on the boundary of R and never in the interior.

③ Take R to be a closed unit disk and $f(0) = 0$.

④ By maximum modulus principle, the maximum value of $|f|$ attains on the boundary.

On l_1, $|f|^2 = \sinh^2 y$

$$\max_{l_1} |f|^2 = \frac{1}{4}(e - e^{-1})^2$$



On l_2, $|f|^2 = \sin^2 x$, $\max_{l_2} |f|^2 = 1$

On l_3, $|f|^2 = \sinh^2 y$, $\max_{l_3} |f|^2 = \frac{1}{4}(e-e^{-1})^2$

On l_4, $|f|^2 = \sin^2 x + \sinh^2(1)$,

$\max_{l_4} |f|^2 = \max_{l_4} \left(\sin^2 x + \sinh^2(1) \right) = \sinh^2(1) + 1$

Therefore, $\max_{R} |f|^2 = \max_{l_4} |f|^2 = \sinh^2(1) + 1$ at $z = \frac{\pi}{2} + i$.

Page 196

(4) $\cos z = -\sin\left(z - \frac{\pi}{2}\right) = -\sum_{n=0}^{\infty} \frac{(-1)^n (z-\pi/2)^{2n+1}}{(2n+1)!} = \sum_{n=0}^{\infty} \frac{(-1)^{n+1} \left(z-\frac{\pi}{2}\right)^{2n+1}}{(2n+1)!}$

Result follows by the unique of taylor series.

(9) $f = \sin(z^2) = \sum_{n=0}^{\infty} \frac{(-1)^n (z^2)^{2n+1}}{(2n+1)!} = \sum_{n=0}^{\infty} \frac{(-1)^n z^{4n+2}}{(2n+1)!}$

Therefore, $f^{(4n)}(0) = f^{(2n+1)}(0) = 0$.

(10) (a) $\sinh z = \frac{e^z - e^{-z}}{2} = \sum_{n=0}^{\infty} \frac{z^{1+2n}}{(1+2n)!}$

$\frac{\sinh z}{z^2} = \sum_{n=0}^{\infty} \frac{z^{-1+2n}}{(1+2n)!} = \sum_{n=0}^{\infty} \frac{z^{2n+1}}{(2n+3)!} + \frac{1}{z}$

(b) It follows from (9).

(11) For $0 < |z| < 4$, $\frac{1}{4z - z^2} = \frac{1}{4z} \left(\frac{1}{1 - z/4} \right) = \frac{1}{4z} \sum_{n=0}^{\infty} \left(\frac{z}{4}\right)^n$

$\Rightarrow \frac{1}{4z - z^2} = \frac{1}{4z} + \sum_{n=0}^{\infty} \frac{z^n}{4^{n+2}}$.